THE

PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, Vol. 134, No. 6B 22 JUNE 1964

Nucleon-Nucleon Potential in Similar Configurations*

YESHWANT R. WAGHMARE
Department of Physics, University of California, San Diego, La Jolla, California
(Received 13 January 1964; revised manuscript received 9 March 1964)

Calculations have been made on pair nuclei in order to obtain an effective interaction for the same shell of each pair. For the pair nuclei O^{18} and Zr^{92} s and d shells are considered, and for the pair nuclei Be^{10} and Ni^{58} the p shell is considered. It is shown that it is impossible to derive a unique effective interaction for the s and d shells due to insufficient information on the low-lying levels of Zr^{92}. However, a Gaussian potential with range $r_0=1.47$ F and a nuclear force strength $v_0=-51$ MeV gives information concerning the effective interaction for the pair nuclei Be^{10} and Ni^{58}.

I. INTRODUCTION

In recent years, several investigations have been made for obtaining information about the nucleon-nucleon potential from nuclear spectroscopy data. Though it is difficult to deduce the exact nature of the interaction between the particles inside a shell-model nucleus, some of the general characteristics of these interactions can easily be brought out. Dawson and Walecka[1] have shown that the nucleon-nucleon scattering data can reproduce the observed bound-state properties of a nucleus, e.g., binding energy, magnetic moment, low-lying energy level spectrum, etc., satisfactorily. The other outstanding feature of these interactions that has been brought out recently, on the basis of simple shell-model calculations, in the framework of the method of relative coordinates,[2–4] is that of the existence of a hard core.[5] Purely considering the level spectrum of oxygen isotopes, Pandya[5] has shown that the level spectrum of these nuclei can be well fitted by a sum of the potentials with (i) $V_0=-300$ MeV, $\lambda=0.5$; (ii) $V_0=+575$ MeV, $\lambda=0.32$, where V_0 is the strength of the singlet potential and λ is its range. However, in these calculations, the triplet forces are assumed negligible. It would thus be interesting to know the nature of the interaction which would operate in the shells having the same orbital quantum number but different energies. The explicit calculations based on such an analysis would certainly provide valuable information on the nucleon-nucleon potential in $T=1$ isotopic spin states. Though such a potential cannot represent in a simple way the K matrix in Brueckner theory,[6] it does elicit the nature of the realistic potential that might exist between the nucleons.

In order to understand the nature of the effective two-body interaction in the same l shells, we present below, in a formal way, the analysis on p, d, and s shells. Section II contains the method that one generally adopts in making calculations of such types. In Sec. III, we present the results on d and s shells. It would be worthwhile to remark that the ordering of the single-particle levels, namely, $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$, is similar in Zr^{92} and O^{18}. It would thus be plausible to make a detailed analysis of one of these nuclei and then apply the results to the other. Similarly, in Sec. IV we analyze the energy levels of Be^{10} and Ni^{58}. Both these nuclei have the ground-state configuration $(p_{3/2})^2$. It is of interest to see that the single-particle energy difference $p_{1/2}-p_{3/2}$ entering in the calculations of Be^{10} energy levels is as yet not established. However, recent calculations of

* Work supported in part by the U. S. Atomic Energy Commission.

[1] J. F. Dawson and J. D. Walecka, Ann. Phys. (N. Y.) 22, 133 (1963).
[2] R. D. Lawson and Maria Goeppert-Mayer, Phys. Rev. 117, 174 (1960).
[3] M. Moshinsky, Nucl. Phys. 13, 104 (1959).
[4] A. N. Mitra and S. P. Pandya, Nucl. Phys. 20, 455 (1960).
[5] S. P. Pandya, Nucl. Phys. 43, 636 (1963).
[6] See, e. g., K. A. Brueckner, J. L. Gammel, and H. Weitzner, Phys. Rev. 110, 431 (1958).

Dawson and Walecka[1] show that this difference is of the order of 5–6 MeV in which case, in accordance with Brueckner theory, such a state would not contribute to the energy of the ground state. We can thus neglect the effect of configuration mixing in Be^{10} while evaluating the strengths and range of the singlet and triplet potentials. These results when applied to Ni^{58} would then give us the wave functions for the low-lying levels in this nucleus. Finally, in the last section (V) we summarize all the results of Secs. III and IV in a coherent way and compare them with the results obtained by other authors.

II. METHOD OF CALCULATIONS

The method of evaluating the matrix elements of a two-body Hamiltonian is straightforward and is well illustrated in the paper of Shah and Pandya.[7] We sketch it briefly for our purpose. The wave function in the jj coupling scheme can be transformed to the LS coupling scheme by means of $9j$ symbols, in the following way

$$|j_1j_2JM\rangle=\sum_{LS} A\begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ L & S & J\end{Bmatrix}|l_1l_2(L),\tfrac{1}{2}\tfrac{1}{2}(S);JM\rangle. \quad (1)$$

This in turn can be transformed into the relative and center-of-mass coordinates by means of Moshinsky brackets[3] as

$$|l_1l_2(L),\tfrac{1}{2}\tfrac{1}{2}(S);JM\rangle = \sum_{\substack{nl\\NL'}} B_{nlNL'}{}^{n_1l_1n_2l_2}|lL'(L),\tfrac{1}{2}\tfrac{1}{2}(S);JM\rangle. \quad (2)$$

Combining (1) and (2), the matrix elements for the central force can be evaluated in a simple way and one obtains,[5]

$$\langle j_1'j_2'JM|H_{12}|j_1j_2JM\rangle = aa'\sum A\begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ L & S & J\end{Bmatrix} A\begin{Bmatrix} l_1' & \frac{1}{2} & j_1' \\ l_2' & \frac{1}{2} & j_2' \\ L & S & J\end{Bmatrix} B_{nlNL'}{}^{n_1l_1n_22} \times B_{nlNL'}{}^{n_1'l_1'n_2'l_2'}[1+(-)^{S+l}]^2 I_{nl}, \quad (3)$$

where

$$I_{nl}=\langle nl\|V(r)\|nl\rangle=\int_0^\infty R_{nl}^2(r)V(r)r^2dr, \quad (4)$$

and $a=a'=\frac{1}{2}$ if the particles are identical, otherwise $(\frac{1}{2})^{1/2}$. $V(r)$ can be chosen to be of the Gaussian form, namely, $V(r)=V_0e^{-(r/r_0)^2}$ and for $R_{nl}(r)$ one can take the harmonic oscillator wave functions. The integrals I_{nl} are extremely useful for the analysis of low-lying nuclear energy levels and we tabulate them in Table I. In what follows, if one assumes a two-body interaction of the type

$$H_{12}=(a'+b'\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)V(r), \quad (5)$$

one can choose a set of parameters a', b', and $\lambda=r_0/r_l$, where r_0 is the range of the Gaussian potential $V(r)$ and r_l the range of the harmonic oscillator wave function $R_{nl}(r)$, that gives reasonably good agreement with the observed results. Knowing the value of r_l and λ, r_0 can be fixed. From expression (5) it is also clear that one can obtain the strength of singlet ($S=0$) and triplet ($S=1$) forces (for $T=1$) as

$$\begin{aligned} V_0'&=V_0(a'-3b')=(a-3b), \quad S=0\\ V_1'&=V_0(a'+b')=(a+b), \quad S=1. \end{aligned} \quad (6)$$

These can be related to the coefficients A_{TS} (which we have used for comparison with other authors) in the following way:

$$\begin{aligned} A_{10}&=V_0'/V_0,\\ A_{11}&=V_1'/V_0. \end{aligned} \quad (7)$$

TABLE I. The matrix elements $I_{nl}=\langle nl|e^{-(r/r_0)^2}|nl\rangle$.

I_{nl} \ λ	1.0	0.9	0.8	0.7	0.5
I_{0s}	0.3536	0.2994	0.2436	0.1886	0.0894
I_{1s}	0.2210	0.1970	0.1730	0.1478	0.0894
I_{2s}	0.1721	0.1542	0.1366	0.1190	0.0798
I_{3s}	0.1500	0.1348	0.1193	0.1036	0.0711
I_{4s}	0.1352	0.1208	0.1092	0.0903	0.0631
I_{0p}	0.1768	0.1340	0.0950	0.0620	0.0179
I_{1p}	0.1547	0.1291	0.1028	0.0765	0.0520
I_{2p}	0.1359	0.1165	0.0967	0.0766	0.0354
I_{3p}	0.1227	0.1063	0.0895	0.0740	0.0387
I_{0d}	0.0884	0.0599	0.0370	0.0204	0.0036
I_{1d}	0.0994	0.0761	0.0540	0.0344	0.0083
I_{2d}	0.0988	0.0796	0.0603	0.0418	0.0121
I_{3d}	0.0981	0.0803	0.0645	0.0447	0.0163
I_{0f}	0.0442	0.0268	0.0145	0.0067	0.0007
I_{1f}	0.0608	0.0422	0.0264	0.0143	0.0021
I_{2f}	0.0669	0.0498	0.0337	0.0207	0.0038
I_{3f}	0.0646	0.0510	0.0338	0.0238	0.0029
I_{0g}	0.0221	0.0120	0.0057	0.0022	0.0001
I_{1g}	0.0359	0.0178	0.0124	0.0057	0.0005
I_{2g}	0.0447	0.0302	0.0189	0.0093	0.0016

III. ANALYSIS OF THE *d-s* SHELL CONFIGURATIONS

Several authors have made detailed calculations of the energy levels of O^{18}. These authors have used different interactions and have obtained results which show varying degrees of agreement with experiments. The choice of this nucleus for the analysis in a way convenient to these authors is also apparent, e.g., Moszkowski[8] has shown that the s states ($l=0$) only can give a qualitative agreement for O^{18} level spectrum; while Dawson, Talmi, and Walecka[9] have neglected the

[7] S. K. Shah and S. P. Pandya, Nucl. Phys. 38, 420 (1962).

[8] S. A. Moszkowski, in *Proceedings of the International Conference on Nuclear Structure, Kingston*, edited by D. A. Bromley and E. W. Vogt (The University of Toronto Press, Toronto, Canada, 1960), p. 502.

[9] J. F. Dawson, I. Talmi, and J. D. Walecka, Ann. Phys. (N. Y.) 18, 330 (1962).

configuration mixing arising from various single-particle states in this nucleus. However, we in our analysis would include all the states (l even and l odd) and also take into account the interactions due to the excited levels. The level spectrum that one would observe in this nucleus is

$$(d_{5/2})^2 \quad J=0,2,4 \qquad (s_{1/2})^2 \quad J=0 \qquad (d_{5/2}s_{1/2}) \quad J=2,3. \tag{8}$$

The effect of the $d_{3/2}$ state on these levels, which lies at $\sim$5 MeV in O^{17}, we do not consider, and the single-particle level separation between $d_{5/2}$ and $s_{1/2}$ we assume to be 0.88 MeV as observed in O^{17}. The constants a and b in Eq. (5) can then be evaluated for various values of λ from the known spacing of 0, 2, and 4 levels of O^{18}, and a set of these constants can then be chosen which would give a best fit with all the observed levels of this nucleus. It is observed that such a set, namely,

$$a=-30.0 \text{ MeV}, \quad b=3.5 \text{ MeV}, \quad \lambda=0.8, \tag{9}$$

gives the values of the above levels (9) as shown in Table II. It is clear that the agreement is good. The strengths of the potentials V_0' and V_i' are then,

$$V_0'=-40.5 \text{ MeV}, \quad V_1'=26.5 \text{ MeV}. \tag{10}$$

If, for the sake of comparison with the results obtained by other authors (Table III), we fix our value of A_{10} as 0.60, then we have for the triplet potential and the strength of the Gaussian potentials, the following values,

$$A_{10}=0.60, \quad A_{11}=0.40, \quad V_0=-68 \text{ MeV}. \tag{11}$$

We remark that the above interaction is quite different from the Rosenfeld[10] or Elliott and Flowers[11] interaction, but compares favorably with that of Barker.[12]

We now proceed to consider the energy levels of Zr^{92}, within the framework of the configuration space mentioned above, which is somewhat larger than in O^{18}. The case of Zr^{92}, is also important from the following point of view. The single-particle states involved in describing the low-lying energy levels of Zr^{92} may be selected from the observed[13] level spectrum of Zr^{91}. If in the spirit of Brueckner theory we include only the near-degenerate configurations of Zr^{92}, and define the near-degeneracy as all configurations within 2.5 MeV of the ground-state configuration, we select the following configuration space for describing the low levels of Zr^{92}:

$$\begin{aligned} (d_{5/2})^2 &\quad J=0,2,4, \\ (d_{5/2}S_{1/2}) &\quad J=2,3, \\ (d_{5/2}d_{3/2}) &\quad J=1,2,3,4, \\ (S_{1/2})^2 &\quad J=0, \\ (d_{5/2}g_{7/2}) &\quad J=1,2,3,4,5,6. \end{aligned} \tag{12}$$

We shall remark on the implications of the extra configurations later.

TABLE II. Calculated and observed levels of O^{18}.

J	0^+	2^+	4^+	0^+	2^+
E_{cal}	G.S.	2.05	3.55	3.60	3.95
E_{exp}	G.S.	1.98	3.55	3.63	3.92

TABLE III. Table of comparison of various parameters with different authors.

Authors	Mass number	A_{10}	A_{11}	V_0 in MeV	r_0 in fermis	Radial shape	Reference No.
Thankappan, Waghmare, and Pandya	90	0.6	0.22	−51	2.10	Gaussian	21
Raz and French	43	0.6	0.20	−30	2.70	Gaussian	22
Elliott and Flowers	18	0.7	−0.26	−48.3	1.4	Yukawa	11
Barker	16	0.5	0.38	−77.3	1.4	Yukawa	12
		0.6	0.60	−95.4	1.4	Yukawa	
True and Ford	206	0.6	0	−54.1	2.65	Gaussian	20
Kearsley	206	0.6	−0.34	−68.8	1.37	Yukawa	19
Band, Kharitonov, and Sliv	206	0.6	0.26	−60.0	2.0	Gaussian	23
Peaslee	16	0.34	0	−60.0	1.4	Yukawa	18
Rosenfeld	16	0.6	−0.33	−35.6	1.4	Yukawa	10
Ours	d, s shells	0.6	0.40	−68	1.47	Gaussian	···
	p shells	0.6	0.08	−51	1.47	Gaussian	···

[10] L. Rosenfeld, *Nuclear Forces* (North-Holland Publishing Company, Amsterdam, 1948).
[11] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A242**, 57 (1957).
[12] F. C. Barker, Phys. Rev. **122**, 572 (1961).
[13] H. J. Martin, Jr., M. B. Sampson, and R. L. Preston, Phys. Rev. **125**, 94 (1962).

For the purpose of calculations, we consider separately the singlet and triplet interactions. The energy levels of Zr^{92} calculated for a Serber force of strength -40 MeV have been published earlier[14] and we discuss them in brief. We plot the energy levels of Zr^{92} as a function of λ (Fig. 1). From expressions (7) and (9) one would obtain the value of r_0 as

$$r_0 = 1.47 \text{ F} \tag{13}$$

and consequently the λ corresponding to Zr^{92} would be 0.57. For this value of λ, the lowest levels 2^+ and 4^+ are predicted rather high compared to the experimental values. It is also clear that a better agreement can be obtained with singlet forces alone for $\lambda \approx 0.50$. In this case, we may identify the 2.06-MeV state as the 2^+ state, 2.90-MeV state to be a close doublet of 3^+, 0^+, and similarly perhaps the 3.28-MeV state also to be a close doublet of 2^+ and 4^+ states. This level scheme, however, would not explain the 1.86-MeV state. One may be tempted to remark that this state might arise due to the excitation of the two protons in the $p_{1/2}$ shells.

One can now introduce the triplet odd forces. It was noticed that the effect is most predominant for the lowest 2^+ and 4^+ states. These states are depressed, and consequently, with suitable choice of triplet forces it may be possible to obtain a reasonable agreement with the experimental results for singlet forces of longer range, i.e., larger value of λ. In any case, $V_1' = -27$ MeV and $\lambda = 0.58$ does not seem to be the best choice for obtaining a good agreement with the experiments. We have at this stage not made a more elaborate analysis (perhaps it would be outside the subject matter of the present paper) since we feel that for this purpose the higher energy levels (beyond the lowest three 0^+, 2^+, 4^+ states) and their spins and parities should be well established by experiments. For example, the spin of the 2.06-MeV state should be useful, since by suitable choice of triplet forces one can predict the second excited 2^+ state near either 2.1 MeV or near 2.9 MeV. Further, the theory predicts the 2.91- and the 3.28-MeV states to be degenerate multiplets of 0^+, 3^+, 2^+, and 4^+ states. It should then be possible to obtain more reliable information on the nature of the singlet and triplet interactions in Zr^{92}, and to compare them with those in O^{18}. We finally remark that the extra configurations in (12) do not have substantial effect on the configurations described in (8) as the off-diagonal elements between various states are relatively very weak.

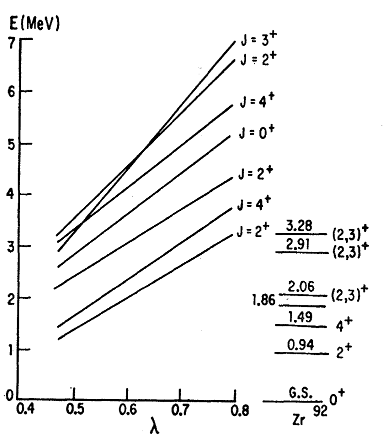

FIG. 1. Calculated and observed energy levels of Zr^{92}.

IV. INTERACTIONS IN p SHELL

The situation as far as the p shell is concerned is different from the one we treated in the d–s shells in the previous section. In what follows we had a set of parameters to be fitted with a variety of levels of O^{18} and

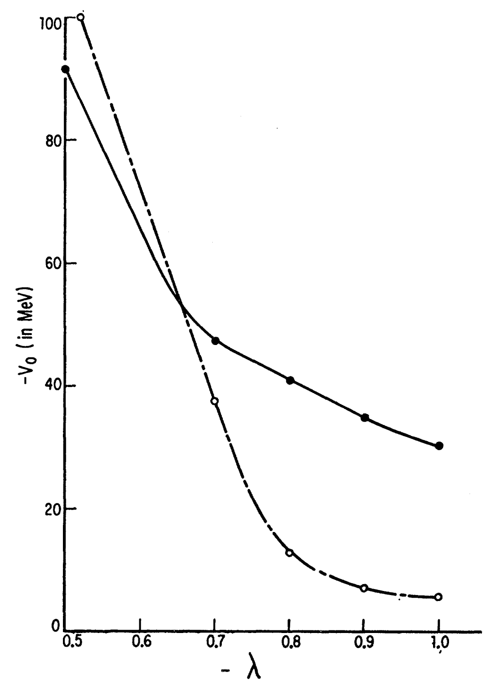

FIG. 2. Variation of $-V_0'$ (solid line) and V_1' (dashed line) with λ, as determined from the analysis of Be^{10} ground-state configuration.

Zr^{92} nuclei, and as such it would not be unreasonable if we obtain only a qualitative picture of the nucleon-nucleon interaction in the absence of some valuable data. The effect of configuration mixing is also quite predominant as far as the low-lying excited levels of these nuclei are concerned. On the other hand, it is interesting to note that the ground-state configuration of Be^{10} can be assumed to be almost pure. It is thus possible to derive a variety of interaction parameters from the observed splitting of the $2^+ - 0^+$ levels of this nucleus and the pairing energy in the ground state. The pairing energy is given by

$$\text{P.E.} = -\langle (j)^2 : J=0 | H_{12} | (j)^2 : J=0 \rangle . \tag{14}$$

[14] Y. R. Wahgmare, Physica 28, 957 (1962).

The observed pairing energy of Be^{10} is 6.10 MeV and with this value of the pairing energy one obtains the values of the parameters V_0' and V_1' as shown in Fig. 2. It is clear from the figure that the triplet potential falls off much rapidly as compared to the singlet potential as the range of the effective two-body force increases. However, as the values of these parameters are derived from the pure p-shell data, namely (i) the 2^+-0^+ separation of the $(p_{3/2})^2$ ground-state configuration and (ii) the pairing energy in the ground state, it is impossible to choose a unique set of these values. On the other hand, from the expression for r_l, it is clear that the radial extension of the harmonic oscillator wave function for Be^{10} is related to that of the Ni^{58} by the relation

$$(r_l)_{\mathrm{Be}^{10}} \approx 0.7 (r_l)_{\mathrm{Ni}^{58}}. \tag{15}$$

If then with the parameters of Be^{10} we calculate the splittings of the 2^+ and 0^+ levels of Ni^{58} using the appropriate values of V_0' and V_i' corresponding to a definite λ we would obtain a set of values as shown in the following table (Table IV). One would be certainly tempted to choose the value of λ as 0.5–0.7. These values of λ would place the corresponding λ for Be^{10} at 0.7–1.0 and one would have a range of values of V_0' and V_1'. Thus it is obvious that in such complex cases it would be difficult to select a set of parameters in this way. There is also another difficulty. If the values of λ are chosen for the two nuclei according to the relation (15), the values of V_0' and V_1' corresponding to each of these λ's are different for the two cases. However, it is clear that this procedure cannot help us in our decision, also due to the fact if we fix the values of V_0' and V_1' it would not reproduce the pairing energy of the Be^{10} nucleus correctly. In what follows, we assume the range of the nucleon-nucleon potential derived from the analysis of the spectrum of d–s shell nuclei, as $r_0 = 1.47$ F. This is a reasonable assumption as according to many-body theory r_0 is not expected to change anywhere inside the nuclear system. With this value of r_0, we obtain the parameters for Be^{10} as

$$\begin{aligned} \lambda &\approx 1.0, \\ V_0' &= -30.64 \text{ MeV}, \\ V_1' &= -3.90 \text{ MeV}. \end{aligned} \tag{16}$$

With these values of V_0' and V_1' (and of course for $\lambda = 0.7$) we obtain the separation of the 2^+-0^+ levels as

$$\Delta = E(2^+) - E(0^+) = 1.15 \text{ MeV}.$$

This value of Δ is smaller than the observed value by 0.3 MeV. However, this is not surprising as we have entirely neglected the effect of configuration mixing in this nucleus. It should be remembered that the single-particle level $f_{5/2}$ in Ni^{57} has as yet not been well established. However, preliminary calculations[15] made on the basis of S-state interactions suggest it to be at $\sim$0.9 MeV as has been suspected[16] by an experimental investigation. If we assume this result, it is obvious that the ground state of Ni^{58} cannot be pure. It is also observed that the off-diagonal matrix elements are rather strong, particularly for the two 0^+ states as compared to the other 2^+ states. It would thus shift Δ to the required value.

TABLE IV. Calculated $\Delta = 2^+ - 0^+$ separation of Ni^{58}. In the last column the observed value of Δ is presented.

λ	1.0	0.9	0.8	0.7	0.6	$\cdots$
Δ in MeV	2.22	2.10	1.95	1.50	1.40	1.45

It is worthwhile to remark that it is possible to obtain a set of parameters that would explain the perturbed levels of Ni^{58} in accordance with

$$\begin{pmatrix} \langle H_{11} \rangle & \langle H_{12} \rangle \\ \langle H_{21} \rangle & \langle H_{22} \rangle \end{pmatrix}$$

and the unperturbed levels of Be^{10}. However, it would certainly complicate matters not only for the evaluation of a particular set of parameters but also in that the forces thus deduced would not explain the behavior of the effective interaction prevailing in the p shell. It should also be mentioned that we cannot derive a similar set of parameters for the $(T=1)\mathbf{p}_{1/2} \times \mathbf{j}$ configuration due to similar reasons. An extensive analysis of two-particle and three-particle $p_{1/2}$ doublets has recently been published.[17] It is thus clear that the nucleon-nucleon interaction operating in the p shell can be defined by the parameters

$$\begin{aligned} A_{10} &= 0.6, \\ A_{11} &= 0.08, \\ -V_0 &= 51 \text{ MeV}. \end{aligned} \tag{17}$$

It is to be noted that the coefficient A_{11} is almost negligible. This can be compared with the result obtained by Peaslee.[18]

V. DISCUSSION AND CONCLUSION

In this section we summarize our results obtained in the previous sections and try to compare them with those obtained by other authors (Table III). Actually such a comparison is quite limited due to the fact that the parameters are evaluated by analyzing the data suitable for the problem at hand. However, from Table III it is clear that the interactions deduced by Elliott and Flowers,[11] Kearsley[19] and Rosenfeld[10] differ in their character markedly from the rest of the inter-

[15] Y. R. Waghmare, R. K. Gupta, and N. Kumar, Progr. Theoret. Phys. (to be published).

[16] M. H. MacFarlane, B. J. Raz, J. L. Yntema, and B. Zeidman, Phys. Rev. **127**, 204 (1962).

[17] P. C. Sood and Y. R. Waghmare, Nucl. Phys. **46**, 18 (1963).

[18] D. C. Peaslee, Phys. Rev. **124**, 839 (1961).

[19] M. J. Kearsley, Phys. Rev. **106**, 389 (1957).

actions. Whereas the Peaslee[18] and True and Ford[20] interactions do not have any triplet component, the interactions determined by TWP,[21] Raz and French[22] and BKS[23] vary from 0.2 to 0.4. It must however be remembered that these interactions have been derived from various available data such as nuclear energy levels, transition probabilities, magnetic moments and stripping reactions. Though it is obvious that any of these properties must be satisfactorily explained by a given set of parameters, due to the approximations that are involved in determining these properties (and the insufficient knowledge about the nucleon-nucleon potential), the situation becomes complicated. In other words, forms of interactions are different as one goes from one property of the nucleus to another, which is not at all surprising. It should also be mentioned that while the parameters of Barker[12] and Peaslee[18] have been determined from the analysis of $p_{1/2}$ and $s_{1/2}$ doublets in the $A\approx 16$ region which would not involve any configuration mixing as far as the $\frac{1}{2}\times\mathbf{j}$ doublets are concerned, the effect of admixtures has been quite predominant as far as the quantitative agreement of the positions of the energy levels are concerned. The analysis of our work in Secs. III and IV differs from the rest of the authors in two ways: (1) while the configuration mixing is entirely neglected by Dawson, Talmi, and Walecka,[9] the triplet forces are entirely neglected by Peaslee[18] and True and Ford.[20] (2) The nature of the interaction is assumed the same in all the configurations. It has however been indicated by Thankappan, Waghmare, and Pandya[21] that the two-body effective interaction in Zr^{90} is configuration-dependent. This is more evident from our present analysis where we take into account both the singlet as well as triplet forces and the effect of configuration mixing as well. In view of the calculations on the many-body systems, such an effect may not be observed in Be^{10}. However, it is certainly important in the case of Ni^{58} where the first excited state in Ni^{57} lies close to the ground state. It is thus clear that the interactions that we have derived in subsequent sections determine the nature of the effective nucleon-nucleon potential. It is also clear that it is not possible, at this stage, to get such an information about the d–s shells.

[20] W. W. True and K. W. Ford, Phys. Rev. **109**, 1675 (1958).
[21] V. K. Thankappan, Y. R. Waghmare, and S. P. Pandya, Progr. Theoret. Phys. (Kyoto) **26**, 22 (1961).
[22] B. J. Raz and J. B. French, Phys. Rev **104**, 1411 (1956).
[23] I. M. Band, Yu I. Kharitonov, and L. A. Sliv, Nucl. Phys. **35**, 136 (1962).

ACKNOWLEDGMENTS

The author is grateful to Professor Maria Geoppert Mayer and Dr. S. P. Pandya for many stimulating discussions. The author expresses his gratitude to Professor Keith A. Brueckner for the hospitality extended to him at the University of California, La Jolla.

PHYSICAL REVIEW VOLUME 134, NUMBER 6B 22 JUNE 1964

Velocity-Dependent Potentials and the Shell Model of Oxygen-18

Bruce H. J. McKellar
Daily Telegraph Theoretical Department, School of Physics, University of Sydney, Sydney, Australia
(Received 14 February 1964)

An expansion of the shell-model matrix elements of the velocity-dependent potential $m^{-1}[p^2V(r)+V(r)p^2]$ in the Talmi integrals of V is derived and applied to calculate the energy levels of O^{18} using the nucleon-nucleon potential of Green. It is found that the correct ordering of the levels is obtained but the potential must be altered slightly to obtain agreement comparable with that given by Dawson, Talmi, and Walecka using the Brueckner-Gammel-Thaler potential.

1. INTRODUCTION

THE possibility that velocity-dependent potentials (v.d.p.) could replace the hard core of the nucleon-nucleon potential, permitting more tractable calculations in many-body problems, was suggested by Peierls[1] at the Kingston Conference. It has since been discussed by many authors.[2]

Green's calculations are the most extensive, and they have been supplemented by Preston, Armstrong, and Bhaduri. The phase-shift data were fitted quite well, although the agreement obtained is probably not the best possible. The triplet odd parameters, in particular, could be readjusted with advantage. The potential used by these authors was of the form

$$-V(r)+m^{-1}(p^2\omega(r)+\omega(r)p^2),$$

[1] R. E. Peierls, *Proceedings of the International Conference on Nuclear Structure, Kingston, 1960*, edited by D. A. Bromley and E. W. Vogt (North-Holland Publishing Company, Amsterdam, 1960), p. 7.
[2] M. Razavy, G. Field, and J. S. Levinger, Phys. Rev. **125**, 269 (1962); O. Rojo and L. M. Simmons, *ibid.* **125**, 273 (1962); A. M. Green, Nucl. Phys. **33**, 218 (1962); M. A. Preston, P. J. Armstrong, and R. K. Bhaduri, Phys. Letters **2**, 183 (1962); E. Werner, Nucl. Phys. **35**, 324 (1962); F. Peischl and E. Werner, *ibid.* **43**, 372 (1963).